851. An Examination of Allen's Empirical Bond-energy Scheme, and its Application to Paraffins and Cycloalkanes, Olefins, Alkyl Alcohols and Bromides, and Amines.

By H. A. Skinner.

The relationship of the empirical bond-energy scheme proposed recently by Allen to the L.C.B.O. molecular-orbital treatment of saturated paraffins used by Brown is pointed out.

Allen's scheme is shown to correlate very well with the heats of formation of gaseous paraffins, from methane to the nonanes, provided that the steric interactions are calculated by the methods here proposed. The scheme is extended to cover mono-olefins $(C_2 - C_7)$ as well as a number of cycloalkanes and cycloalkenes. An application of the scheme to the limited data on heats of formation for alcohols, alkyl bromides, and alkylamines is also presented.

1. Introduction.—The bond-energy scheme recently proposed by Allen¹ correlates very well with the experimental heats of formation of gaseous paraffin hydrocarbons from methane to the heptanes. McCullough and Good² have applied Allen's scheme also to a number of alkanethiols and alkyl sulphides and disulphides: they found that the correlation is remarkably good, the average deviation between calculated and experimental heats of formation being less than the average experimental uncertainty interval.

However, Allen's scheme is less satisfactory when applied to highly branched paraffins containing 8 or more C atoms; e.g., the calculated and the experimental heats of formation of gaseous 2,3,3-trimethylpentane, 2,2,3,3-tetramethylbutane, and 2,2,3,3- and 2,3,3,4tetramethylpentane differ by 2.16, 3.10, 4.33, and 3.3 kcal./mole, respectively. These deviations are well outside the experimental uncertainty intervals.

A modification of Allen's scheme described by Skinner³ fits the experimental data for the octanes and nonanes reasonably well, but needs a larger number of empirical parameters than does Allen's scheme. The present paper describes a version which retains the main features of Allen's scheme and differs from it only in the method of evaluating steric hindrances.

2. Theoretical Basis.—A basis for Allen's scheme lies in the molecular-orbital treatment of paraffin hydrocarbons given by Brown⁴ in 1953, in which the molecular orbitals were represented as linear combinations of bond-orbitals (the L.C.B.O. approximation). The method may be illustrated by reference to methane: the molecular orbitals are written $\psi_i = \sum_{i=1}^{4} c_i \phi_i$, where the C-H bond orbitals, ϕ_i , need not be specified precisely. The

energies of the orbitals ψ_i are obtained by solution of the secular determinant:

³ Skinner, Anales real Soc. españ. Fís. Quím., 1960, 56, B, 931.

⁴ Brown, J., 1953, 2615.

¹ Allen, J. Chem. Phys., 1959, **31**, 1039. ² McCullough and Good, J. Phys. Chem., 1961, **65**, 1430.

where α = coulomb parameter for the CH bond-orbital, β = resonance integral for interaction between two adjacent CH bond-orbitals, and S = overlap integral of adjacent CH bond-orbitals. Writing $\gamma = \beta - S\alpha$, we obtain the roots of expression (1) as:

$$E_1 = \alpha + 3\gamma/(1+3S); \qquad E_2 = E_3 = E_4 = \alpha - \gamma/(1-S);$$

so that the total electronic energy of the eight bonding electrons of methane is

$$2(E_1 + E_2 + E_3 + E_4) = 8\alpha + 6\gamma[(1 + 3S)^{-1} - (1 - S)^{-1}]$$

= $8\alpha - 24\gamma S + 48\gamma S^2$ - (terms in higher powers of S).

To extend the treatment to ethane and higher paraffins, additional coulombic and resonance integrals are needed, *viz*.:

Coulomb	parameters	Resonance	integrals	Overlap	integrals
CH	-α	CH : CH	β	CH : CH	S
CC	$\alpha + h\gamma$	CH : CC	θβ	CH : CC	θS
		CC : CC	$\eta \beta$	CC : CC	ηS

Brown derived a general formula for the total electronic energy of the bonding electrons of a paraffin hydrocarbon, $C_n H_{2n+2}$, in terms of these parameters, as a power series in S:

$$E = (6n + 2)\alpha + (2n - 2)h\gamma - 4S\gamma(b_1 + b_2\theta^2 + b_3\eta^2) + 2\gamma S^2\{[(3n - 5)b_2 - 3b'_2]h\theta^2 + [(3n - 7)b_3 - 3b'_3]h\eta^2 + 6(c_1 + c_2\theta^2 + c_3\eta\theta^2 + c_4\eta^3)\} + (\text{terms in higher powers of } S). (2)$$

In equation (2), b_1 , b_2 , and b_3 are the numbers of adjacent pairs of bonds, and c_1 , c_2 , c_3 , c_4 the numbers of adjacent trios of bonds in the molecule C_nH_{2n+2} , of the types shown below:

$$\begin{array}{cccccccccc} C < H & C < C & C < C & H - C < H & H - C < C & H - C < C & C & C - C < C \\ b_1 & b_2 & b_3 & c_1 & c_2 & c_3 & c_4 \end{array}$$

 b'_2 and b'_3 are defined by $b'_2 = \sum A_r$, and $b'_3 = \sum B_r$, where A_r , B_r , are properties of the r-th C-C bond, the summations being over all C-C bonds in the molecule C_nH_{2n+2} . A_r is the number of adjacent pairs C < C C not involving r, and B_r the corresponding number C < C C pairs not involving r.

To shorten equation (2), we make the following substitutions:

and recast equation (2) in the form:

$$E = (2n+2)E_1 + (n-1)E_2 + b_1P_1 + b_2P_2 + b_3P_3 + c_1T_1 + c_2T_2 + c_3T_2 + c_4T_4 + x_1X_1 + x_2X_2.$$
 (3)

The scheme is applied to the paraffins C_1 to C_5 in the annexed Table.

The bonding-energy equation (3) can, however, be considerably simplified by introducing the parameters:

$$\begin{split} B_1 &= E_1 + \frac{3}{2}P_1 + T_1; \\ B_2 &= E_2 - 3P_1 + 6P_2 - 4T_1 + 6T_2 + 6X_1; \\ \Gamma &= P_1 - 2P_2 + P_3 + 2(T_1 - 2T_2 + T_3 - X_1 + X_2); \\ \Delta &= -T_1 + 3T_2 - 3T_3 + T_4. \end{split}$$

Paraffin	(2n + 2)	(n - 1)	b_1	b_2	b_3	c_1	C.2	c_{3}	C4	x_1	x_2
С	4	0	6	0	Ō	4	0	Õ	Ō	Ō	ō
CC	6	1	6	6	0	2	6	0	0	6	0
CCC	8	2	7	10	1	2	8	2	0	10	2
CCCC	10	3	8	14	2	2	10	4	0	14	4
с сс с	10	3	9	12	3	3	9	3	1	12	6
CCCCC	12	4	9	18	3	2	12	6	0	18	6
CCCC C	12	4	10	16	4	3	11	5	1	16	8
с ссс	12	4	12	12	6	4	12	0	4	12	12

The equations for the paraffins C_1 to C_5 given above, expressed in terms of these four parameters, are then as tabulated.

	B_1	B_2	г	Δ
С	4	0	0	0
CC	6	1	0	0
CCC	8	2	1	0
CCCC	10	3	2	0
с сс с	10	3	3	1
CCCCC	12	4	3	0
cccc	12	4	4	1
	12	4	6	4

For the general case of a paraffin, $C_n H_{2n+2}$, equation (3) transforms into

$$E = (2n+2)B_1 + (n-1)B_2 + b_3\Gamma + c_4\Delta,$$
(4)

which is the same as Allen's formula, except for the omission of corrections for steric repulsion.

3. Corrections for Steric Repulsion.—Allen modified equation (4) when applied to isopentane and branched paraffins containing six or more carbon atoms to allow for steric repulsions in these molecules. The gauche configuration of n-butane is known to be less stable than the *trans*-form by *ca.* 0.5 kcal./mole, and may be regarded as sterically weakened by the close approach (to within *ca.* 2 Å) of two hydrogen atoms attached to the

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1,4-carbon atoms. Allen therefore subtracted from equation (4) a term 0.5m, where m is the number of pairs of gauche 1,4-C-H bonds present in the molecule C_nH_{2n+2} .

However, the magnitude of a gauche 1,4 steric repulsion appears to be more severe than 0.5 kcal./mole in certain cases; e.g., in alkylcyclohexanes, the experimental value is ca. 0.95 kcal./mole per 1,4-interaction.⁵ We propose therefore to distinguish between different types of 1,4-interaction, the basis for distinction being that some interactions are more "rigid" than others.

The steric repulsion between 1,4-H atoms could be relieved either by twisting the interfering CH_3 groups from "staggered" conformations, or by widening the C-C-C angles of the chain connecting the groups. In Fig. 1, we have represented schematically four types of steric interaction, labelled S_{11} , S_{12} , S_{13} , and S_{22} , which differ in respect of their ability to make use of the twisting release mechanism.





The interaction S_{11} could be relieved somewhat by a slight rotation of one or both of the 1,4-CH₃ groups from the staggered towards the eclipsed conformation: only a small angle of "twist" can be considered as a mechanism of release, since the rotational-energy barrier is substantial. Interactions S_{12} and S_{13} are also open to release by twisting, but in both these cases only one of the C-H bonds of an interacting pair brings release on twisting. On the other hand, the interaction S_{22} (and also S_{32} , not shown in Fig. 1) cannot effectively be reduced by slight twisting; *e.g.*, in 2,2,3-trimethylbutane, twisting of terminal CH₃ to reduce S_{22} simultaneously increases S_{12} , so that no net advantage accrues. Hence we accept that $S_{22} = S_{23} > S_{21} = S_{31} > S_{11}$, following the order of ability to use the "twisting" mechanism of release.

The "angle " release mechanism, represented schematically in Fig. 2, may be classified according to the angles θ_i , θ_j . Angle release is limited by the resistance of θ_i , θ_j to deformation: we assume that angle rigidity decreases in the order $\theta_3 > \theta_2 > \theta_1$, and is greater

⁵ Prosen, Johnson, and Rossini, J. Res. Nat. Bur. Stand., 1947, 39, 173.

when more than one C-C-C angle of a given carbon atom is involved in repulsion interactions, e.g., $\theta_2^2 > \theta_2$.

Accordingly, we propose to characterize 1,4-interactions in respect both of twisting and angle release possibilities, e.g., $S_{11}(\theta_1, \theta_2)$, $S_{12}(\theta_1, \theta_2^2)$: we represent these more simply as $S_{11}(12)$ and $S_{12}(12^2)$.

4. Application to Paraffins.—Equation (4), with steric interaction terms added, can be tested against extensive thermochemical data for paraffin hydrocarbons.⁶ Strictly, heats of atomization at 0° K, corrected to apply to molecules without zero-point energy, should be used for the test.⁷ In practice, the difficulty of evaluating accurately the zeropoint energies of all but the simplest of polyatomic molecules renders it impossible to carry out an extensive test in this way: we have used the experimental data appropriate to 25°c. Consequently, the success of the scheme, despite some theoretical justification, rests in part on an empirical approach.

The parameters B_1 and B_2 (equation 4) may be evaluated from the experimental heats of atomization of methane and ethane, giving $B_1 = 99.29$ kcal./mole and $B_2 = 78.84$ kcal./mole. On substitution of these values, equation (4) may be transformed into the more convenient form, *viz*.:

$$\Delta H^{0}_{f}(C_{n}H_{2n+2}, g) = \Delta H^{0}_{f}(CH_{4}, g) + (n-1)[\Delta H^{0}_{f}(C_{2}H_{6}, g) - \Delta H^{0}_{f}(CH_{4}, g)] - b_{3}\Gamma - c_{4}\Delta + \text{steric terms}; \quad (5)$$

or,
$$\Delta H^0_f(C_n H_{2n+2}, g) = -15.54 - 2.35n - b_3 \Gamma - c_4 \Delta + [S],$$
 (6)

where [S] measures the total steric repulsion correction in $C_n H_{2n+2}$.

Experimental heats of formation of paraffins ^{6,8} are compared with values calculated from equation (6) in Table 1. The parameters Γ and Δ were given the values (in kcal./mole) $\Gamma = 2.58$; $\Delta = -0.55$. The values chosen for the steric 1,4-interactions are:

	S11	S_{12}	S_{22}		S11	S_{12}	S22		S11	S12	S_{22}
(12)	0.33	0.38	0.43	$(2^{n}2^{n})$	0.75	0.80	1.00	$(3^{m}3^{m})$	0.85	0.90	1.25
(12^{n})	0.48	0.53	0.58	$(2^n 3^m)$	0.80	0.85	1.10	(22)	0.45	0.50	0.70
(13 ^m)	0.52	0.57	0.62	, , , , , , , , , , , , , , , , , , ,							
. ,				(n = 2 or)	r3; m =	= 2, 3, 4	, 5, or 6.)			

The agreement between the calculated and the experimental ΔH^{0}_{f} values is in general excellent, and within the limits of error of determination of the heats of combustion: the deviations are largest for neopentane and 2,3-dimethylpentane, and there may be grounds for re-measuring the heats of combustion of both these compounds.

For 2,2,4-trimethylpentane, and also for 2,2,3,4- and 2,2,4,4-tetramethylpentane, the maintenance of the staggered configuration about each C-C bond results in a prohibitively close H · · · H approach, owing to 1,5-interaction. It seems probable that these molecules will avoid this by twisting away from the staggered towards the eclipsed configuration about one or more of the C-C bonds: as a consequence we cannot calculate the steric repulsion in these cases by the method used for the other hydrocarbons in Table 1. To obtain agreement with the experimental ΔH^0_t values, the terms [S] are, empirically, 4.02 kcal./mole (2,3,3-trimethylpentane), 7.71 kcal./mole (2,2,3,4-tetramethylpentane), and 8.00 kcal./mole (2,2,4,4-tetramethylpentane).

5. Application to Mono-olefins.—The bond-bond interaction scheme can be applied to olefins by dividing the interactions into those between σ -bonds, and those involving π - or "quasi- π "-bonds, and summing the contributions from both.

The σ -bond interactions include those already described for paraffins, and additional terms involving the sp^2 - or trigonal carbon atoms, C*. We represent the bond energy and

⁶ A.P.I. Tables, Project 44, Carnegie Inst. Technology, Pittsburg, 1953-61.

⁷ Cottrell, J., 1948, 1448.

⁸ Labbauf, Greenshields, and Rossini, J. Chem. Eng. Data, 1960, 6, 261.

TABLE 1.

			$-\Delta H^0$	$-\Delta H^{0}$	
Compound	Steric terms	[S]	(calc.)	(obs.)	Diff.
Propane			25.17	24.82	0.35
Butane	—	<u> </u>	30.10	30.15	-0.05
Isobutane		<u> </u>	32.13	32.15	-0.02
Pentane	_		35.03	35.00	0.03
Isopentane	$S_{11}(12)$	0.33	36.73	36.92	-0.19
Neopentane	- 11()		40.57	39.67	0.90
Hexane	<u> </u>		39.96	39.96	0.00
2-Methylpentane	$S_{11}(12)$	0.33	41.66	41.66	0.00
3-Methylpentane	$2S_{11}(12^2)$	0.96	41.03	41.02	0.01
2.2-Dimethylbutane	$2S_{10}(13^2)$	1.14	44.36	44.35	0.01
2.3-Dimethylbutane	$2S_{11}(2^22^2)$	1.50	42.52	42.49	0.03
Heptane			44.89	44.89	0.00
2-Methylhexane	$S_{11}(12)$	0.33	46.59	46.60	-0.01
3-Methylhexane	$2S_{11}(12^2)$	0.96	45.96	45.96	0.00
3-Ethylpentane	$3S_{11}(12^3)$	1.44	45.48	45.34	0.14
2.2-Dimethylpentane	$2S_{10}(13^2)$	1.14	49.29	49.29	0.00
2.3-Dimethylpentane	$S_{12}(2^23^3) + S_{12}(12^3) + S_{12}(2^22^3)$	2.08	46.87	47.62	-0.75
2.4-Dimethylpentane	$2S_{11}(-2)$ (-2) (-2)	0.66	48.29	48.30	-0.01
3.3-Dimethylpentane	$4S_{-1}(134)$	2.28	48.15	48.17	-0.02
2.2.3-Trimethylbutane	$2S_{12}(2^23^3) + 2S_{22}(2^23^3)$	3.90	48.56	48.96	-0.40
Octane			49.82	49.82	0.00
2-Methylheptane	S.,(12)	0.33	51.52	51.50	0.02
3-Methylheptane	$2S_{-1}(12^2)$	0.96	50.89	50.82	0.07
4-Methylheptane	$2S_{11}(12^2)$	0.96	50.89	50.69	0.20
3-Ethylhexane	$3S_{11}(12^3)$	1.44	50.41	50.40	0.01
2.2-Dimethylbexane	$2S_{11}(13^2)$	1.14	54.22	53.71	0.51
2 3-Dimethylbexane	$S_{12}(2^22^3) + S_{12}(12^3) + S_{12}(2^22^3)$	2.28	51.65	51.13	0.52
2 4-Dimethylhexane	$S_{11}(12) + 2S_{12}(12^2)$	1.39	52.49	52.44	0.05
2.5-Dimethylhexane	$2S_{11}(12)$ $(12)_{12}(12)$	0.66	53.22	53.21	0.01
3.3-Dimethylhexane	$4S_{\rm ex}(13^4)$	2.28	53.08	52.61	0.47
3 4-Dimethylhexane	$2S_{12}(12^3) + 2S_{12}(2^32^3)$	2.66	51.22	50.91	0.31
3-Ethyl-2-methylpentane	$2S_{12}(12^3) + 2S_{12}(2^22^3) + S_{12}(12^3)$	3.24	50.64	50.48	0.16
3-Ethyl-3-methylpentane	$3S_{12}(13^6) + 3S_{12}(13^6)$	3.57	51.79	51.38	0.41
2 2 3-Trimethylpentane	$S_{12}(10^{\circ}) + S_{22}(10^{\circ}) + S_{12}(2^{3}3^{3}) + S_{12}(2^{3}3^{3})$	4.43	52.06	52.61	0.26
2,2,0 minietiny ipentane	$+ S_{ac}(2^33^3)$	1 10	02 00	02 01	0.00
2.2.4-Trimethylpentane	see text	[4.02]	<u> </u>	53.37	
2,3,3-Trimethylpentane	$S_{12}(2^{2}3^{5}) + S_{22}(13^{5}) + S_{22}(2^{2}3^{5}) + S_{23}(13^{5}) + 2S_{22}(2^{2}3^{5})$	5.39	52.00	51.73	0.27
2.3.4-Trimethylpentane	$S_{22}(2^22^3) + S_{22}(2^22^3) + 2S_{22}(2^22^3) + S_{22}(2^22^3)$	4.15	51.76	51.97	-0.21
2.2.3.3-Tetramethylbutane	(3^33^3)	7.50	53.40	53.99	-0.59
Nonane			54.75	54.74	0.01
3 3-Diethylpentane	85(134)	4.96	55.33	55.44	
2 2 3 3-Tetramethyl-	$2S_{22}(13^5) + 6S_{22}(3^33^5)$	8.74	57.09	56.70	0.30
nentane	$25_{23}(10) + 55_{23}(00)$	071	01 00	00.10	0.99
2 2 3 4-Tetramethyl-	see text	[7.7]]	_	56.64	
pentane	Sou toxt	[]		00.04	
2.2.4.4-Tetramethyl-	see text	[8·00]	_	57.83	
pentane	SOU VORU	[0 00]		01.00	
2.3.3.4-Tetramethyl-	$2S_{10}(2^23^5) + 6S_{20}(2^23^5)$	8.30	56.05	56.46	0.41
pentane				50 10	
r					

interaction terms involving trigonal carbon atoms (occurring in olefins and alkyl free radicals) by the following symbols:

	E 1*	= energy of	of an i	solated	C*-H b	ono	d
	E_{2}^{*}	= energy of	of an i	solated	С *- С Ъ	ond	1
	E_{2}^{**}	= energy of	of an i	solated	C*-C*	bon	ıd
P 1 *	= inte	raction bet	ween	the adja	icent pa	air	$C^*\!\!<^H_H$
P2*		,,	,,	,,	,,	,,	$C^*\!\!<^H_C$
P ₂ **		,,	,,	,,	,,	"	$C^*\!\!<^H_{C^*}$



The problem may be simplified by introducing the following composite parameters: †

$$\begin{split} B_1^* &= E_1^* + P_1^* + \frac{1}{3}T_1^* \\ B_2^* &= E_2^* - \frac{3}{2}P_1 + 3P_2 - P_1^* + 2P_2^* - 2T_1 + 3T_2 - \frac{2}{3}T_1^* + T_2^* \\ B_2^{**} &= E_2^{**} - 2P_1^* + 4P_2^{**} - \frac{4}{3}T_1^* + 2T_2^{**} \\ \Gamma^* &= (P_1^* - 2P_2^* + P_3^*) + (T_1^* - 2T_2^* + T_3^*) \\ \Delta^* &= -T_1^* + 3T_2^* - 3T_3^* + T_4^* \\ \Gamma^{**} &= (P_1^* - P_2^* - P_2^{**} + P_3^{**}) + (T_1^* - 2T_2^{**} + T_3^*) \\ \Delta^{**} &= -T_1^* + 3T_2^{**} - 3T_3^* + T_4^{**} \end{split}$$

With the aid of these parameters, the σ -contribution to the total bonding energy of an olefin can be expressed by a sum of the relevant "bond-energy terms" $(B_1, B_2, B_1^*, B_2^*, B_2^{**})$ and bond-interactions $(\Gamma, \Gamma^*, \Gamma^{**}, \Delta, \Delta^{**})$. The energy equations can be written down very simply from inspection of the molecular formulæ, by counting the number of bonds and C-C-C interactions, *e.g.*:

	B_1	B_2	$B_1 *$	B_2 *	B ₂ **	г	Δ	г*	Γ**	Δ**
CH,=CH,	0	0	4	0	1	0	0	0	0	0
CH, CH=CH,	3	0	3	1	1	0	0	0	1	0
$C_2 H_{5} \cdot CH = CH_2 \dots$	5	1	3	1	1	1	0	0	1	0
(CH ₃) ₂ C=CH ₂	6	0	2	2	1	0	0	1	2	1

 \dagger Terms corresponding to X_1 , X_2 (equation 3) have been omitted in these expressions.

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The π -contribution to the total bonding energy includes the energy of the π -component of the C=C bond itself, and the π -hyperconjugation energy arising from the interaction of saturated groups attached to the double-bond with the π -electrons of the double bond. As a simplifying measure, we assume that a grouping R-C=C (where R = alkyl) contributes a quantity h_{π} to the total hyperconjugation energy. The π -contribution in an olefin is thus expressed by $(x_{\pi} + mh_{\pi})$, where x_{π} is the contribution in ethylene, and m is the number of C-C* bonds present in the olefin. Accordingly, this scheme gives, for the total bonding energy of an olefin C_nH_{2n+1} ·CH=CH₂:

$$E = (2n+1)B_1 + (n+1)B_2 + 3B_1^* + B_2^* + B_2^{**} + b_3\Gamma + c_4\Delta + \Gamma^{**} + x_\pi + h_\pi;$$
(7)

and, for an olefin $(C_nH_{2n+1})(C_mH_{2m+1})C=CH_2$:

$$E = 2(n + m + 1)B_1 + (n + m - 2)B_2 + 2B_1^* + 2B_2^* + B_2^{**} + b_3\Gamma + c_4\Delta + \Gamma^* + 2\Gamma^{**} + \Delta^{**} + x_{\pi} + 2h_{\pi}.$$
 (8)

As for paraffins, these equations can be transformed to enable heats of formation to be calculated directly; thus, for example, equation (7) transforms into:

$$\Delta H^{0}_{f}(C_{n}H_{2n+1} \cdot CH = CH_{2}, g) = \Delta H^{0}_{f}(C_{2}H_{4}, g) + n[\Delta H^{0}_{f}(C_{2}H_{6}, g) - \Delta H^{0}_{f}(CH_{4}, g)] -b_{3}\Gamma - c_{4}\Delta - \Gamma^{**} - h_{\pi} - [B_{1} - B_{1}^{*} + B_{2}^{*} - B_{2}]; \quad (9)$$

or, on substitution of the experimental ΔH^0_f values for C_2H_4 , C_2H_6 , and CH_4 , and $\Gamma = 2.58$, $\Delta = -0.55$ (as in paraffins), into:

$$\Delta H^{0}_{f}(g) = 12.50 - 2.35n - 2.58b_{3} + 0.55c_{4} - A, \qquad (10)$$

where $A = \Gamma^{**} + h_{\pi} + (B_1 - B_1^*) + (B_2^* - B_2)$. If the values A = 5.27, $(\Gamma^* + \Delta^{**}) = 1.30$ kcal./mole, are chosen, the following equations apply to olefins R¹CH=CH₂, R¹R²C=CH₂, R¹HC=CHR², R¹R²C=CHR³, and $\mathbb{R}^{1}\mathbb{R}^{2}\mathbb{C}=\mathbb{C}\mathbb{R}^{3}\mathbb{R}^{4}$ {the steric corrections (omitted so far) are represented by the term [S]}:

- (i) $R^1CH=CH_2$ ($R^1 = C_nH_{2n+1}$) $\Delta H^{0}_{t}(g) = 7 \cdot 23 - 2 \cdot 35n - 2 \cdot 58b_{3} + 0 \cdot 55c_{4} + [S];$ (11)
- (ii) $R^1R^2C=CH_2$ ($R^2 = C_mH_{2m+1}$) $\Delta H^{0}_{f}(g) = 0.66 - 2.35(n+m) - 2.58b_{2} + 0.55c_{4} + [S];$ (12)
- (iii) R¹CH=CHR² $\Delta H^{0}_{f}(g) = 1.96 - 2.35(n+m) - 2.58b_{3} + 0.55c_{4} + [S];$ (13)

(iv)
$$R^{1}R^{2}C=CHR^{3}$$
 ($R^{3} = C_{p}H_{2p+1}$)
 $\Delta H^{0}_{f}(g) = -4.61 - (n + m + p) 2.35 - 2.58b_{3} + 0.55c_{4} + [S];$ (14)

(v)
$$R^{1}R^{2}C = CR^{3}R^{4}$$
 ($R^{4} = C_{q}H_{2q+1}$)
 $\Delta H^{0}_{f}(g) = -11 \cdot 18 - (n + m + p + q) 2 \cdot 35 - 2 \cdot 58b_{3} + 0 \cdot 55c_{4} + [S].$ (15)

The steric corrections in an olefin may involve terms additional to those already met in paraffins. One of the most important of these is the repulsion between alkyl groups attached cis to the double bond. We represent these generally by S_{cis} , and distinguish between different types according to the angles θ_i , θ_j (for angles R-C= and R'-C=, respectively). For an angle C-C=C we have θ_1 in $\underset{H}{\overset{C}{\to}}$ C=C (i = 1) and θ_2 in $\underset{C}{\overset{C}{\to}}$ C=C (i = 2). Provided neither R nor R' is tertiary, the repulsion S_{cit} is considered to be due to the close

approach (1,4) of hydrogen atoms across the double bond. To some extent, repulsion might be alleviated by angle-widening and by displacements from planarity of the olefinic bonds: in either event, release should be most effective in the case $S_{cis}(11)$, and become increasingly difficult in the more crowded situations represented by $S_{cis}(12)$, $S_{cis}(12^2)$, and $S_{cis}(2^22^2)$.



Empirically, we propose the values:

 $S_{cis}(11) = 1.0$ kcal./mole; $S_{cis}(12) = 1.4$ kcal./mole; $S_{cis}(12^2) = 1.6$ kcal./mole; $S_{cis}(2^22^2) = 2.2$ kcal./mole.

When one or more of the interacting *cis*-groups is tertiary, the steric overcrowding becomes very severe owing to the closeness of approach of the 1,5-carbon atoms. The steric interference is larger than in any of the cases discussed above, and from specific examples quoted in Table 2 the following values are indicated:

$$S_{cis}(11, \operatorname{Bu}^{t} \cdots R) = 5 \cdot 8 \text{ kcal./mole};$$

$$S_{cis}(12, \operatorname{Bu}^{t} \cdots R) = 6 \cdot 5 \text{ kcal./mole};$$

$$S_{cis}(11, \operatorname{Bu}^{t} \cdots \operatorname{Bu}^{t}) = 14 \cdot 5 \text{ kcal./mole}.$$

In addition to S_{cis} , there are two further steric interactions involving the double bond. These are: (1) The repulsion between 1,4-gauche C-H bonds of a C₄ chain which includes one olefinic carbon atom, e.g., (A). These interactions (represented by S^*) are comparable with the 1,4-gauche interactions in paraffins, but may be weaker because of the widening of the angle 2,3,4 relative to the regular tetrahedral angle. To estimate S^* , we have assumed $S^*(i 2) = S(i 2) - 0.10$. (2) The repulsion between 1,4-gauche C-H bonds in



a C_4 chain which includes both olefinic carbon atoms, e.g., (B). This interaction involves three hydrogen atoms, represented by $S_3^{**}(ijk)$. The magnitude of S_3^{**} should be of the order of twice the interaction $S_{22}(ij)$ in paraffins. For k = 0 (*i.e.*, in alk-1-enes) this expectation is reasonably well borne out: when k = 1 (*i.e.*, in alk-2- or -3-enes), the S_3^{**} values appear to be larger. The S_3^{**} values used in Table 2 were derived by assuming $S_3^{**}(ij0) = 2S_{22}(ij)$, and $S_3^{**}(ij1) = 2S_{22}(ij) + 0.7$ kcal./mole.

Table 2 gives a comparison of $\Delta H^{0}_{f}(g)$ values for olefins as calculated from equations (11)-(15) with experimental values given in the A.P.I. Tables,⁶ by Bartolo and Rossini,⁹ and by Rockenfeller and Rossini.¹⁰ The measure of agreement is not as impressive as in the case of the paraffins (Table 1); nevertheless, in most examples there is agreement within the uncertainty of experimental measurement of the heats of combustion, and only in five cases (cis-hex-2-ene, 2,3-dimethylbut-1-ene, 3-methyl-trans-hex-3-ene, 4,4-dimethylpent-1-ene, 2-ethyl-3-methylbut-1-ene) does the divergence become serious, reaching values of the order 1 kcal./mole.

6. Application to Cycloalkanes and Cycloalkenes.—The application of the bond-bond

 ⁹ Bartolo and Rossini, J. Phys. Chem., 1960, 64, 1685.
 ¹⁰ Rockenfeller and Rossini, J. Phys. Chem., 1961, 65, 267.

	TABLE 2.				
			$-\Delta H^{0}_{f}$	$-\Delta H^{0}_{f}$	
Compound	Steric terms	[S]	(calc.)	(obs.)	Diff.
Propene			-4.88	-4.88	0.00
But-1-ene			0.05	0.03	0.02
cis-But-2-ene	S(11)	1.00	1.74	1.67	0.07
trans-But-2-ene			2.74	2.67	0.07
2-Methylpropene		_	4.04	4.04	0.00
Pent-1-ene		<u> </u>	4.98	5.00	-0.02
cis-Pent-2-ene	S(11)	1.00	6.67	6.71	-0.04
trans-Pent-2-ene	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~		7.67	7.59	0.08
2-Methylbut-1-ene	$S_{11}^{*}(12)$	0.23	8.74	8.68	0.06
3-Methylbut-1-ene	-11 ()		7.01	6.92	0.09
2-Methylbut-2-ene	$S_{\rm sig}(12)$	1.40	10.26	10.17	0.09
Hex-1-ene	~ === /		9.91	9.96	-0.05
cis-Hex-2-ene	S(11)	1.00	11.60	12.51	-0.91
trans-Hex-2-ene	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~		12.60	12.88	-0.28
cis-Hex-3-ene	S(11)	1.00	11.60	11.38	+0.22
tyans-Hex-3-ene	-en()		12.60	13.01	-0.41
2-Methylpent-1-ene	S*(12)	0.23	13.67	14.19	-0.52
3-Methylpent-1-ene		<u> </u>	11.94	11.82	0.12
A-Methylpent-1-ene			11.94	12.24	-0.30
2-Methylpent-2-ene	$S_{12}(12)$	1.40	15.19	15.98	-0.79
2-Methyl-cis-pent-2-ene	$S_{ers}(12^2) + S_{ers}^*(12^2)$	1.98	14.61	14.86	-0.25
3-Methyl-thews-pent-2 ene	$S_{iii}(12^2) + S_{i1}^{i1}(12^2)$	1.98	14.61	15.08	-0.47
A-Methyl-cis-pent-2-ene	$S_{cis}(12) + S_{11}(12)$	1.00	13.63	13.73	-0.10
4-Methyl-trans-pent-2-ene	Scis(11)		14.63	14.69	-0.06
2-Fthylbut-1-ene	25*(12)	0.46	13.44	13.38	± 0.06
2 3-Dimethylbut-1-ene	$2S_{11}^{(12)}$ $2S_{-}^{*}(2^{2}2)$	1.10	14.83	15.85	-1.02
2,3-Dimetry ibut-1-one		1 10	11.00	14.78 4	± 0.05
3 3-Dimethylbut-1-ene	5-**(310)	0.94	14.51	14.70	-0.19
3,3-Dimenyibut-1-ene	53 (010)	0.01	1101	14.25 4	0.26
9 2-Dimethylbut-9-ene	$25 \cdot (2222)$	4.40	16.18	16.68	-0.50
2, 3- Dimetry ibut-2-ene	20 cis (2 2)	1 10	1010	15.91 4	0.27
Hent-1-ene		_	14.84	14.89	-0.05
3-Methyl-cis-bex-3-ene	$S_{11}(12^2) + S_{11}*(12^2)$	1.98	19.54	19.22	0.32
3-Methyl-trans-hex-3-ene	$S_{eu}(12^2) + S_{11}(12^2)$	1.98	19.54	18.60	0.94
9 4-Dimethylpent-l-ene	$S_{es}(12) = S_{f1}(12)$	0.23	20.63	20.27	0.36
4 4-Dimethylpent-1-ene	$S_{11}^{(12)}$	0.27	20.11	19.20	0.91
2 4-Dimethylpent-2-ene	$S_{11}(01)$	1.40	22.15	21.44	0.71
4 4-Dimethyl-cis-pent-2-ene	$S_{\rm eff}(11)$ Bu ^t -Me)	[5.8]	17.27	17.60	-0.33
4 4-Dimethyl-trans-pent-2-ene	S.**(311)	1.64	21.43	21.46	-0.03
2-Fthyl-3-methylbut-1-ene	$S_{*}^{**}(12^{2}0) + 2S_{**}^{*}(2^{2}2^{2})$	2.56	18.30	19.25	-0.95
2 3 3-Trimethylbut-1-ene	$S_{3}^{**}(3^{3}2^{2}0) + 2S_{3}^{**}(3^{3}2^{2})$	3.70	20.67	20.67	0.00
Oct-l-ene	C_3 (0 2 0) $2C_{12}$ (0 2)		19.77	19.82	-0.05
2 2-Dimethyl-cis-hex-3-ene	$S_{tr}(11 \text{ Bu}^{t}-\text{Et})$	5.8	22.20	21.77	0.43
2.2.Dimethyl-trans-hex-3-ene	S.**(311)	1.64	26.36	26.16	0.20
3-Fthyl-2-methylpent-1-ene	$2S_{-} = (22^2)$	1.10	24.69	24.40	0.29
2 4 4-Trimethylpent-1-ene	2012 (22)	[2.62]	21.00	26.68	
2.4.4-Trimethylpent-2-ene	$S_{\rm ev}(12$ Bu ^t -Me)	[6.5]	25.49	25.50	-0.01
Dec-1-ene		[0 0] —	29.63	29.45	0.18
2 2 5 5-Tetramethyl-cis-hex-3-ene	$S_{\rm ev}(11, {\rm Bu}^{\rm t} - {\rm Bu}^{\rm t})$	[14.5]	28.90	28.92 4	-0.02
2.2.5.5-Tetramethyl-trans-hex-3-ene	2S.**(311)	3.28	20.12	39.43 0	0.69
a,a,o,o i otraniotity: www.ack o one		0 20		00 10	0.00

^a Derived from heats of hydrogenation. ^b ΔH_{vap} values assumed to be the same as for the corresponding paraffins.

interaction scheme to cycloalkanes and cycloalkenes (cyclic paraffins and olefins) is straightforward and involves the interaction parameters which have already been described. There are, however, additional steric terms which need evaluation, the most important being: (1) Ring strain, represented by $S_{\rm R}$. This may arise from angle deformation (small rings), from the forced adoption of eclipsed (or near-eclipsed) conformations (as in cyclopentanes), or from severe overcrowding (big rings). For present purposes, we propose the values (in kcal./mole):

> Cyclohexanes, $S_{\rm R}(6) = 0.0$; Cyclohexenes, $S_{\rm R}(6) = 0.8$; Cyclopentanes, $S_{\rm R}(5) = 6.2$; Cyclopentenes, $S_{\rm R}(5) = 5.0$.

			$-\Delta H^{0}_{f}$	$-\Delta H^{0}$	
Compound	Steric terms	[<i>S</i>]	(calc.)	(obs.)	Diff.
Cyclohexane			29.58	29.43	0.15
Methylcyclohexane			36.54	36.99	-0.45
Ethylcyclohexane	$S_{11}(12)$	0.33	41.14	41.05	0.09
Dimethylcyclohexanes:	II()				
1.1-	$S_{\bullet}(3^{2}1, 3^{2}1)$	1.24	43.74	43.26	0.48
cis-1.2-	$S_{\bullet}^{\bullet}(2^{2}1, 2^{2}2^{2}) + S_{1\bullet}(2^{2}2^{2})$	2.38	40.67	41.15	-0.48
trans-1.2-	$S_{11}(22)$	0.45	43.05	43.02	0.03
cis-1.3-			43.50	44.16	-0.66
trans-1.3-	$S_{2}(2^{2}1, 2^{2}1)$	1.16	42.34	42.20	0.14
cis-1.4-	S. (2 ² 1, 2 ² 1)	1.16	42.34	42.22	0.12
trans-1.4-	-3(,,		43.50	44.12	-0.62
trans-Decahydronaphthalene			42.98	43.54	-0.56
cis-Decahydronaphthalene	$S_4(2^21, 2^22^2, 2^21)$	2.16	40.82	40.45	0.37
Cyclopentane	$S_{\rm P}(5)$	$\overline{6}\cdot\overline{2}$	18.45	18.46	-0.01
Methylcyclopentane	$S_{\mathbf{R}}(5)$	6.2	25.41	25.50	-0.09
1.1-Dimethylcyclopentane	$S_{\mathbf{R}}(5)$	6.2	33.85	33.05	0.80
Dimethylcyclopentanes:					
trans-1,2-	$S_{\rm R}(5)$	$6 \cdot 2$	32.37	32.67	-0.30
cis-1.2-	$S_{\mathbf{R}}(5) + 2S_{\mathbf{n}}(22)$	7.6	30.97	30.96	0.01
trans-1.3-	$S_{\mathbf{R}}(5)$	$6 \cdot 2$	32.37	32.47	-0.10
cis-1.3-	$S_{\mathbf{R}}(5)$	$6 \cdot 2$	32.37	31.93	0.44
Cvclohexene	$S_{\mathbf{R}}(6)$	0.8	1.42	1.28	0.14
1-Methylcyclohexene	$S_{\mathbf{R}}(6)$	0.8	10· 34	10.38	-0.04
1-Ethylcyclohexene	$S_{\rm R}(6) + S_{11}^{*}(12)$	1.03	15.04	15.20	-0.16
Vinylcyclohexane		<u> </u>	11.42	11.55 4	-0.13
Ethylidenecyclohexane	$S_{cir}(12)$	1.40	14.67	15.09 0	-0.42
Cyclopentene	$S_{\rm R}(5)$	5.0	- 7.71	-7.73	0.02
1-Methylcyclopentene	$S_{\mathbf{R}}(5)$	5.0	1.21	1.30	-0.09
3-Methylcyclopentene	$S_{\mathbf{R}}(5)$	5.0	-0.75	-2·07 b	1.32
4-Methylcyclopentene	$S_{\mathbf{R}}(5)$	5.0	-0.75	-3.53 b	2.78
1-Ethylcyclopentene	$S_{\rm B}(5) + S_{11}^{*}(12)$	5.23	5.91	4.91	1.00
Vinylcyclopentane	$S_{\mathbf{R}}(5)$	$6 \cdot 2$	0.29	-0·73 ª	0.92
Allylcyclopentane	$S_{\mathbf{R}}(5)$	6.2	$5 \cdot 2$	5.95 ª	-0.73
Methylenecyclopentane	$S_{\mathbf{R}}(5)$	6.2	-2.68	-2·76 ª	0.08
Ethylidenecyclopentane	$S_{\mathbf{R}}(5)$	6.2	4.94	4·85 ª	0.09

TABLE 3.

^a ΔH_{vap} assumed the same as in the corresponding cycloparaffin. for 1-methylcyclopentene.

 $\Delta H_{\rm vap}$ assumed the same as

(2) The 1,4-interactions of axial methyl (or alkyl) in alkylcyclohexanes. The repulsions



involve three hydrogen atoms, represented by $S_3(i^2j, i^2k)$ (cf. C); these are not open to release by "twisting," and hence we have assumed that $S_3(i^2j, i^2k) = S_{22}(i^2j) + S_{22}(i^2k)$.

A comparison of calculated and experimental ΔH^{0}_{f} values for cyclohexanes, cyclopentanes, cyclopentenes, and cyclohexenes is (C) given in Table 3. The experimental data are from A.P.I. tables⁶

and from recent papers by Labbauf and Rossini,¹¹ and by Speros and Rossini;¹² the calculated ΔH_{f}^{0} were derived from equations (16-20).

Cycloalkanes:

$$\Delta H^{0}_{f}(C_{n}H_{2n}) = -2.35n - 2.58b_{3} + 0.55c_{4} + [S].$$
⁽¹⁶⁾

1-Alkylcyclohexenes:

$$\Delta H^{0}_{t}(C_{6}H_{9} \cdot C_{n}H_{2n+1}) = 13 \cdot 28 - (n+5)2 \cdot 35 - 2 \cdot 58b_{3} + 0 \cdot 55c_{4} + [S].$$
(17)

1-Alkylcyclopentenes:

$$\Delta H^{0}_{f}(C_{5}H_{7} \cdot C_{n}H_{2n+1}) = 13 \cdot 28 - (n+4)2 \cdot 35 - 2 \cdot 58b_{3} + 0 \cdot 55c_{4} + [S].$$
(18)

¹¹ Labbauf and Rossini, J. Phys. Chem., 1961, 65, 476.
 ¹² Speros and Rossini, J. Phys. Chem., 1960, 64, 1723.

3- or 4-Alkylcyclopentenes:

$$\Delta H^{0}_{f}(C_{5}H_{7} \cdot C_{n}H_{2n+1}) = 19.85 - (n+4)2.35 - 2.58b_{3} + 0.55c_{4} + [S].$$
(19)

Decahydronaphthalene:

$$\Delta H^{0}_{f}(C_{10}H_{18}) = -7.96 - 2.58b_{3} + 0.55c_{4} + [S].$$
⁽²⁰⁾

The agreement between observed and calculated ΔH^0_{f} in Table 3 is on the whole good, except in two cases (3- and 4-methylcyclopentene).

7. Application to Substituted Paraffins, $C_nH_{2n+1}X$.—For a substituted paraffin $C_nH_{2n+1}X$, the bond interaction scheme leads to:

$$\Delta H^{0}_{f}(C_{n}H_{2n+1}X, g) = \Delta H^{0}_{f}(CH_{3}X, g) - (n-1)2\cdot35 - 2\cdot58b_{3} - b'_{3}\Gamma_{X} + 0\cdot55c_{4} - c'_{4}\Delta_{X} + [S], \quad (21)$$

$$b'_{3} = \text{number of C-C-X interactions, } P_{3}^{X},$$

$$c'_{n} = \text{number of C-C-X interactions, } T_{n}^{X}$$

where

$$c'_4$$
 = number of C-C<^C_X trios, T_4^X

and $\Gamma_{\mathbf{X}}$, $\Delta_{\mathbf{X}}$ are composite parameters, defined by:

$$\begin{split} \Gamma_{\mathtt{X}} &= P_1 - P_2 - P_2{}^{\mathtt{X}} + P_3{}^{\mathtt{X}} + 2(T_1 - T_2 - T_2{}^{\mathtt{X}} + T_3{}^{\mathtt{X}} - X_1 + X_2) \\ \Delta_{\mathtt{X}} &= -T_1 + 2T_2 + T_2{}^{\mathtt{X}} - T_3 - 2T_3{}^{\mathtt{X}} + T_4{}^{\mathtt{X}} \end{split}$$

In these equations P_2^{X} , P_3^{X} and T_2^{X} , T_3^{X} , T_4^{X} are derived from P_2 , P_3 , and T_2 , T_3 , T_4 on replacing one carbon atom by X.

McCullough and $Good^2$ have tested equation (21) against experimental data for the alkane-thiols (X = SH) and shown that it fits the facts extremely well. In Table 4 the

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	$-\Delta H^{0}_{f}$	$-\Delta H^{0}_{f}$			$-\Delta H^0_f$	$-\Delta H^{0}_{f}$	
Compound *	(calc.)	(obs.)	Diff.	Compound *	(calc.)	(obs.)	Diff.
Methanol ¹³	47.94	47.94	0.00	1-Bromobutane 19	26.06	26.01	0.05
Ethanol ¹³	55.95	56.17	-0.23	2-Bromobutane ¹⁹	28.66	28.66	0.00
Propan-1-ol ¹⁴	60.88	61.07	-0.19	1-Bromo-2-methylpropane 19	28.09	$28 \cdot 4$	-0.31
Propan-2-ol 14	65.11	65.15	-0.04	2-Bromo-2-methylpropane ¹⁷	3 1·89	$31 \cdot 2$	0.69
Butan-1-ol 15	65.81	65.81	0.00	1-Bromopentane ¹⁹	3 0·99	31.13	-0.14
Butan-2-ol 16	67.83	67.88	-0.05	1-Bromoĥexane 19	35.92	35.88	0.04
2-Methylpropan-1-ol ¹⁶	70.05	70.06	-0.01	1-Bromoheptane ¹⁹	40.85	40.69	0.16
2-Methylpropan-2-ol 16	74 ·87	74.87	0.00	l-Bromo-octane ¹⁹	45.78	46.26	-0.48
Bromomethane 17	9.85	9.6	0.25	Methylamine ²⁰	5.15	$5 \cdot 1$	0.05
Bromoethane 18	16.20	15.3	0.90	Ethylamine ²⁰	11.6	11.2	0.40
1-Bromopropane ^{17, 19}	21·13	21.9819	-0.85	Butylamine ²¹	21.5	$22 \cdot 3$	-0.80
1 1		21.1 17	0.03	s-Butylamine ²¹	$24 \cdot 8$	$25 \cdot 2$	-0.40
2-Bromopropane ¹⁹	23.73	$23 \cdot 55$	0.18	t-Butylamine ²¹	28.7	28.7	0.00
	* ~			1			

* Superior numerals denote reference footnotes.

test is made by using the limited experimental data available for alkyl bromides and alcohols, and amines. Equations (22-24) were used to calculate ΔH_{f}^{0} values.

¹³ Green, J. Appl. Chem., 1961, **11**, 397.

¹⁴ Snelson and Skinner, Trans. Faraday Soc., 1961, 57, 212 (ΔH_{vap} from Wadsö, personal communication).

¹⁵ Gundry, Head, and Lewis, Trans. Faraday Soc., 1962, 58, 1309.

- ¹⁶ Skinner and Snelson, Trans. Faraday Soc., 1960. 56. 1776.
- ¹⁷ Skinner, Royal Inst. Chem., Monograph No. 3, 1958.
 ¹⁸ Lane, Linnett, and Oswin, Proc. Roy. Soc., 1953, A, 216, 361.
 ¹⁹ Bjellerup, Acta Chem. Scand., 1961, 15, 231.
- ²⁰ Jaffe, Thesis, Univ. Maryland, 1958.
- ²¹ Evans, Fairbrother, and Skinner, Trans. Faraday Soc., 1959, 55, 399.

4408 Brown: The Preparation, Properties, Structure, and Alcohols ($\Gamma_0 = 5.66, \Delta_0 = 1.43$): $\Delta H^0_t(C_nH_{2n+1}OH, g) = -45.59 - 2.35n - 2.58b_3 - 5.66b_3' + 0.55c_4 + 1.43c_4' + [S].$ (22) Alkyl bromides ($\Gamma_{Br} = 4.0, \Delta_{Br} = 1.4$): $\Delta H^0_t(C_nH_{2n+1}Br, g) = -7.5 - 2.35n - 2.58b_3 - 4.0b_3' + 0.55c_4 + 1.4c_4' + [S].$ (23) Alkylamines ($\Gamma_N = 4.1, \Delta_N = 1.0$): $\Delta H^0_t(C_nH_{2n+1}\cdot NH_2, g) = -2.8 - 2.35n - 2.58b_3 - 4.1b_3' + 0.55c_4 + 1.0c_4' + [S].$ (24) CHEMISTRY DEPARTMENT, UNIVERSITY OF MANCHESTER, MANCHESTER, 13. [Received, April 13th, 1962.]